Figure 6: A brain image with low noise: hard segmentation from the SMS model via “hardening” formulae (2) and (3). Notice how the detailed branching of the gray matter has been successfully resolved by the model.

subsequence of \( (P^n \mid n) \), which after relabelling shall still be denoted by \( (P^n \mid n) \) for convenience, such that

\[
P^n \to P^* \quad \text{in} \quad L^2(\Omega, \mathbb{R}^K), \quad n \to \infty.
\]

Then by the \( L^2 \) lower semi-continuity of Sobolev measures,

\[
9\varepsilon \int_\Omega |\nabla p_i^*|^2 \leq \liminf_{n \to \infty} 9\varepsilon \int_\Omega |\nabla p_i^n|^2, \quad i = 1 : K.
\]

Furthermore, with possibly another round of subsequence refinement, one can assume

\[
P^n(x) \to P^*(x), \quad a.e. \quad x \in \Omega, \quad n \to \infty.
\]

Since the probability simplex \( \Delta_{k-1} \) is closed and \( P^n(x) \in \Delta_{K-1} \), one concludes that

\[
P^*(x) \in \Delta_{K-1}, \quad a.e. \quad x \in \Omega.
\]

And by Fatou’s Lemma [17, 28], one has

\[
\int_\Omega \frac{(p_i^*(1 - p_i^*))^2}{\varepsilon} \leq \liminf_{n \to \infty} \int_\Omega \frac{(p_i^n(1 - p_i^n))^2}{\varepsilon}, \quad i = 1 : K.
\]

(In fact, the equality holds by Lebesgue’s Dominated Convergence [28].)

After the above subsequence selection on \( P^n \)'s, one naturally has an associated subsequence of \( (U^n \mid n) \), which for convenience is still denoted by \( (U^n \mid n) \) after relabelling. For each specific channel \( i \), we then consider two scenarios separately.

Suppose \( p_i^*(x) \equiv 0, a.e. x \in \Omega \). We then define for that channel

\[
u_i^*(x) \equiv 0, \quad x \in \Omega.
\]

Such a channel is called a “dumb” channel.